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Fuzzy sliding mode control for discrete nonlinear systems*

F. Qiao. Q. M. Zhu, A. Winfield and C. Melhuish**

Faculty of Computing, Engineering and Mathematical Sciences
University of the West of England
Frenchay Campus, Coldharbour Lane, Bristol BS16 1QY, UK
Tel: +44 (0)117-344-2533, Fax: +44 (0)117 344 3800
eMail: quan.zhu@uwe.ac.uk

Abstract: Sliding mode control is introduced into classical model free fuzzy logic control for discrete time nonlinear systems with uncertainty to the design of a novel fuzzy sliding mode control to meet the requirement of necessary and sufficient reaching conditions of sliding mode control. The simulation results show that the proposed controller outperforms the original fuzzy sliding mode controller and the classical fuzzy logic controller in stability, convergence and robustness.

Key words: Fuzzy logic, sliding mode control, nonlinear system, discrete time

1. Introduction

The fuzzy logic control (FLC) has been an active research topic in automation and control theory since the work of Mamdani (1974) based on the fuzzy sets theory of Zadeh (1965). The concept of FLC is to utilise the qualitative knowledge of a system to design a practical controller, FLC is generally applicable to plants that are ill-modelled, but qualitative knowledge of experienced operators available for design. It is particularly suitable for those systems with uncertain or complex dynamics. In general, a fuzzy control algorithm consists of a set of heuristic decision rules and can be regarded as a nonmathematical control algorithm, in contrast to a conventional feedback control algorithm. Such a nonmathematical control algorithm has been proved to be very attractive whenever controlled system cannot be well defined or modelled.

The sliding mode control (SMC) was originally developed for variable structure systems in continuous domain. In his survey paper, Utkin (1977) gives a thorough description of the sliding mode theory in continuous time. Also, Slotine and Li (1991) describe continuous sliding mode controllers in detail. Later, the research of discrete time SMC has been attracted more attention, such as Furuta (1990), Gao and Hung (1993) and Golo and Milosavljevic (2000), as for the implementation of the controller on a digital computer requires a certain sampling interval and the assumption of an infinite switching time does not hold anymore.

The principle of sliding mode control is introduced into classical model free fuzzy logic control, this principle provides guidance to design a fuzzy controller for system stability. The combination of the two control principles, called fuzzy sliding mode control (FSMC), provides an alternative to design a robust controller for nonlinear systems with uncertainty (Yu, et al., 1998; Shih and Lu, 1994; Ting, et al. 1996). Here, in this paper, both sufficient and necessary reaching conditions of sliding mode control for discrete nonlinear systems are introduced into the original fuzzy sliding mode control. The simulation results on an inverted pendulum with MATLAB show the proposed FSMC superior to the original fuzzy sliding mode controller.

The remaining of this paper is organised as follow. In Section 2, SMC for both continuous and discrete systems is reviewed. A fuzzy sliding mode controller for discrete time nonlinear systems is developed and presented in the following section. A case study for an inverted pendulum with the proposed FSMC is simulated on MATLAB in Section 4. Conclusions are drawn in the final section.

2. Sliding mode control

The sliding mode control schemes have been widely developed over several decades of years (Slotine and Sastry, 1983; Habibi and Richards, 1992; Fink and Singh, 1998). Essentially, the SMC uses discontinuous control action to drive state trajectories toward a specific hyperplane in the state space, and to maintain the state trajectories sliding on specific hyperplane until the origin of the state space is reached.

In an SMC system, the control commands are adequately designed such that the states will move towards the desired sliding plane. Once the states reach the sliding surface, the system is said to be in

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** Author to whom correspondence should be addressed sliding mode. During the sliding mode, the system possesses some invariance properties, such as robustness, order reduction and disturbance rejection.

The first step to design a sliding mode control is to determine the sliding hyperplane with desired dynamics of the corresponding sliding motion. And the next step is to design the control input so that the state trajectories are driven and attracted toward the sliding hyperplane and then remained sliding on it for all subsequent time.

In the following, the sliding mode control for continuous and discrete time system is reviewed.

2.1 Continuous sliding mode control

For a single input and single output continuous nonlinear system with n state variables, the companion form is as follow,

$$\dot{x}^{(n)}(t) = f(X(t)) + b(X(t))u(t) + d(t) \qquad (2.1a)$$

and

$$y(t) = x(t)\text{, for } t \geq 0 \qquad (2.1b)$$

where the state vector is $X(t) = [x(t), \dot{x}(t), \cdots x^{(n-1)}(t)]^T$, $u(t)$ is the control input, $y(t)$ is the system output and $d(t)$ is an external disturbance. If the reference output is $y_r(t)$, the above dynamic equations can be transferred into the following state equations with error signal $e_1(t) = y(t) - y_r(t)$ and its derivatives as state variables:

$$\begin{aligned} \dot{e}_1(t) &= e_2(t) \\ \dot{e}_2(t) &= e_3(t) \\ &\vdots \\ \dot{e}_n(t) &= f(E(t)) + b(E(t))u(t) + d(t) \end{aligned} \qquad \text{for } t > 0 \text{ .(2.2)}$$

Let $E(t) = [e_1(t), e_2(t), \cdots e_n(t)] \in R$ and $\delta \in R$. A linear functional $s : E \to \delta$ is defined by

$$s(t) = CE(t) \qquad (2.3)$$

where $C = [c_1, c_2, \cdots c_{n-1}, 1]$ (c_i s are all real numbers).

Then, a sliding hyperplane can be represented as $s(t) = 0$ or

$$c_1 e(t)_1 + c_2 e_2(t) + \cdots + e_n(t) = 0 \qquad (2.4)$$

The scalar $s(t)$ of (2.3) is defined as the distance to the $s(t)$ hyperplane of (2.4).

The motion sliding on the sliding hyperplane is commonly referred to as a sliding mode. If the following condition is satisfied with the designed control input, the just mentioned requirements on the state trajectories can be fulfilled:

$$s(t)\dot{s}(t) < 0 \qquad (2.5)$$

where $\dot{s}(t)$ represents the time derivative of $s(t)$. Inequality of (2.5) is called the reaching condition of sliding control for continuous systems under which the state will move toward and reach a sliding surface.

If the control input is so designed that the inequality $s(t)\dot{s}(t) < 0$ is satisfied, together with the properly chosen sliding hyperplane, the state will be driven toward the origin of the state space along the sliding hyperplane from any given initial state. This is the way of the SMC that guarantee asymptotic stability of the systems.

2.2 Discrete sliding mode control

The implementation of the controller on a digital computer requires a certain sampling interval and the assumption of an infinite switching time does not hold anymore. Milosavljevic (1985) derived conditions which ensure the existence of a sliding mode in discrete time, the so-called quasi-sliding mode. Techniques of designing sliding mode controller for discrete time domain were proposed by Drakunov and Utkin (1989) and Furuta (1990). Golo and Milosavljevic (2000) proposed a new design for discrete time SMC. Sira-Ramirez (1991) investigated sliding mode controllers for discrete nonlinear. The systems, however, that can be controlled are subject to strong restrictions.

The discretisation form of (2.1) with first order approximation is as follow

$$\begin{aligned} x_1(k+1) &= x_1(k) + Tx_2(k) \\ x_2(k+1) &= x_2(k) + Tx_3(k) \\ &\vdots \\ x_{n-1}(k+1) &= x_{n-1}(k) + Tx_n(k) \\ x_n(k+1) &= x_n(k) + Tf(X(k)) + Tb(X(k))u(k) + Td(t) \end{aligned} \qquad (2.8a)$$

and

$$y(k) = x_1(k)\text{, for } k = 0, 1T, 2T, \cdots, nT\text{ ,} \qquad (2.8b)$$

(T is the sampling rate)

where $X(k) = [x_1(k), x_2(k), \cdots x_n(k)]^T$.

$s(k)$ is defined similar to that in continuous time domain

$$s(k) = CX(k). \qquad (2.9)$$

where $C = [c_1, c_2, \cdots c_{n-1}, 1]$ (c_i s are all real numbers). $s(k) = 0$ is the so-called hyperplane which is equivalent to the sliding surface in continuous time. Since it cannot be ensured that the states remain on the surface for all time because of the discretisation, it is also called quasi-sliding mode. Again, C is chosen such that the system is asymptotically stable while being on the hyperplane.

One way to evaluate C is to use the pole placement method. The system has to be written in its controllability canonical form and the final state can be represented in terms of remaining when staying on the sliding surface S .

In discrete case, discretisation form of (2.5) will be

$$s(k)[s(k+1)-s(k)]<0 \tag{2.10}$$

This is the necessary reaching condition for the sliding mode control, it only makes the sliding motion toward the sliding surface.

In order to satisfy the reaching, the following inequality has to be satisfied to meet the requirement of the sufficient reaching condition for the existence of a sliding motion, too

$$\left|s(k+1)\right|<\left|s(k)\right| \tag{2.11}$$

The combination of (2.10) and (2.11) can make sure the sliding motion convergent.

Just as in continuous time, in discrete time, equivalent control $u_{eq}(k)$ is designed to try to keep the system on sliding surface which is represented as

$$s(k+1)=s(k) \tag{2.12}$$

and the switching term, $u_s(k)$, is designed to (satisfy the reaching condition (2.10) and (2.11)) overcome the external disturbance and reaching the stable state.

However, the most disadvantage of using SMC is the chattering phenomenon. Because a discontinuous switching control is applied to the plant, chattering always appears as a source to excite the unmodelled high frequency dynamics of the controlled system. One commonly used method to eliminate the chattering is to replace the relay control by a saturating approximation. Another method is to apply fuzzy logic control to the SMC system such that a smooth and reasonable hitting control can be generated to reduce the chattering.

3. Fuzzy sliding mode control

It is difficult to design a sliding mode controller for nonlinear system. Up to now, there have been quite a lot of research on the combination of sliding mode control with fuzzy logic control techniques for improving the robustness and performance of nonlinear systems with uncertainty (Chen and Chang, 1998; Li and Shieh, 2000; Wang et al, 2001, a, b; and Chang et al. 2002).

The purposed fuzzy control is called fuzzy sliding mode control (FSMC) as it is based on the principle of SMC. Here, an FSMC for discrete nonlinear systems is proposed.

For the discrete time system (2.8), assume that the sliding surface is

$$s(k)=\sum_{i=1}^{n} c_i x_i(k) \tag{3.1}$$

where $c_i>0, i=1,2,\cdots,n$. And we can get

So from (2.10), the necessary reaching condition is as follow

$$\begin{aligned} & s(k)(s(k+1)-s(k)) \\ & =\sum_{i=1}^{n} c_i x_i(k)\left(\sum_{i=1}^{n} c_i x_i(k+1)-\sum_{i=1}^{n} c_i x_i(k)\right) \\ & =\sum_{i=1}^{n} c_i x_i(k)(\sum_{i=1}^{n} c_i x_{i+1}(k)+ \\ & \qquad c_n f(X(k))+c_n b(X(k))u(k)+c_n d(k))T<0 \end{aligned} \tag{3.2}$$

Suppose that c_n is positive and $b(X)$ is negative, (3.2) presents that as $s(k)$ is positive, increasing $u(k)$ will result in decreasing $s(k)(s(k+1)-s(k))$, and that as $s(k)$ is negative, decreasing $u(k)$ will result in decreasing $s(k)(s(k+1)-s(k))$. On the other hand, suppose $c_n b(X)$ is positive, (3.2) denotes that decreasing $u(k)$ will cause decreasing $s(k)(s(k+1)-s(k))$ as $s(k)$ is positive, and increasing $u(k)$ will cause decreasing $s(k)(s(k+1)-s(k))$ as $s(k)$ is negative. Therefore, all such statements above can be summarised as a fuzzy rule base used in FLC. (The design of a fuzzy sliding mode controller is detailed in Section 4.)

Note that the control law should meet the requirement for the sufficient reaching condition (2.11) too.

4. FSMC for inverted pendulum

4.1 System description

The inverted pendulum is often used as a benchmark for all kinds of controllers. It is a nonlinear, unstable system which makes it challenge to control. The system is composed of a rigid pole and a cart on which the pole is hinged to the cart through a pivot such that it has only one degree of freedom. The goal of the control is to make the pole upright. The dynamic model equations for the plant are

$$M\ddot{x}+N=u \tag{4.1}$$

$$N=m\ddot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta \tag{4.2}$$

$$P-mg=-ml(\ddot{\theta}\sin\theta+\dot{\theta}^2\cos\theta) \tag{4.3}$$

$$I\ddot{\theta} = Pl\sin\theta - Nl\cos\theta \tag{4.4}$$

where M is the mass of the cart, m is the mass of the of the pendulum, $I = (1/3)ml^2$ is the moment of the inertia of the pendulum, θ is the angular position of the pendulum deviated from the equilibrium position, x is the position of the cart, l is the half length of the pendulum. The system friction is omitted for simplicity.

The dynamic equation of θ can be rewritten as

$$\begin{aligned}&\left[(M+m)(ml^2+I)-(ml\cos\theta)^2\right]\ddot{\theta}+\\&(ml\dot{\theta})^2\cos\theta\sin\theta-\\&(M+m)ml\sin\theta+ml\cos\theta u=0\end{aligned} \tag{4.5}$$

If we define $x_1 = \theta$ and $x_2 = \dot{\theta}$, then the first order approximation of the plant plus zero order hold for the sampling representative of (4.5) can be expressed as

$$\begin{aligned}x_1(k+1) &= x_1(k) + Tx_2(k)\\ x_2(k+1) &= x_2(k) + T\left[\frac{(M+m)m\lg\sin x_1(k)}{(M+m)(ml^2+I)-(ml\cos x_1(k))^2}\right.\\ &\quad -\frac{(mlx_1(k))^2\cos x_1(k)\sin x_1(k)}{(M+m)(ml^2+I)-(ml\cos x_1(k))^2}\\ &\quad \left.-\frac{ml\cos x_1(k)u(k)}{(M+m)(ml^2+I)-(ml\cos x_1(k))^2}\right]\end{aligned} \tag{4.6}$$

(T is the sampling rate).

4.2 Controller design

The expression for the linear functional $s(k)$ is chosen as

$$s(k) = c_1x_1(k) + x_2(k) \tag{4.7}$$

The corresponding sliding hyperplane is represented by

$$s(k) = c_1x_1(k) + x_2(k) = 0 \tag{4.8}$$

Review the necessary and sufficient reaching conditions,

$$s(k)(s(k+1) - s(k)) < 0 \tag{4.9}$$

and

$$|s(k+1)| < |s(k)| \tag{4.10}$$

S, ΔS, $\Delta|S|$ and ΔU are the fuzzy variables of $s(k)$, $(s(k+1)-s(k))$, $|s(k+1)|-|s(k)|$ and control variable increment Δu, respectively. The membership functions for S, ΔS, $\Delta|S|$ and ΔU are chosen to be in the shape of triangular type.

From (4.6), we can know

$$b = -T\left[\frac{ml\cos x_1(k)}{(M+m)(ml^2+I)-(ml\cos x_1(k))^2}\right] < 0 \quad ,$$

for $-\frac{\pi}{2} < x_1(k) < \frac{\pi}{2}$

which is the same as assumption in (3.2). Then the fuzzy rules described in Section 3 are further detailed as follow

1) If $s(k) > 0$ and $-s(k) < s(k+1) < s(k)$, increasing $u(k)$ will strengthen (4.9) and meet the sufficient condition (4.10).

2) If $s(k) > 0$ and $s(k+1) > s(k)$, greatly increasing $u(k)$ will be helpful to reach the conditions of (4.9) and (4.10).

3) If $s(k) > 0$ and $s(k+1) < -s(k)$, reasonably decreasing $u(k)$ will be helpful to meet the sufficient reaching requirement of (4.10).

4) If $s(k) < 0$ and $s(k) < s(k+1) < -s(k)$, decreasing $u(k)$ will strengthen (4.9) and meet the sufficient condition (4.10).

5) If $s(k) < 0$ and $s(k+1) > -s(k)$, greatly increasing $u(k)$ will be helpful to meet (4.10).

6) If $s(k) < 0$ and $s(k+1) < s(k)$, reasonably decreasing $u(k)$ will be helpful to meet (4.9) and (4.10).

4.3 Simulation results

The parameters of the inverted pendulum for simulation are $l = 0.5m$, $M = 2kg$ and $m = 0.3kg$. And the coefficient c_1 is chosen as $c_1 = 6$. The initial conditions of the inverted pendulum are supposed to be: (a) $\theta = -1$ and $\omega = 1$, and (b) $\theta = 0.5$ and $\omega = 1$. At $t = 2.5$ second, an external disturbance with $d(t) = 20$ is added to the pendulum to verify the robustness of the controller. The simulation is made on MATLAB, and the simulation results with the proposed FSMC (PFSMC) are shown on Figure 4.1 (a) and (b), on which the simulation results with the original FSMC (OFSMC) and a classical fuzzy logic control (FCL) with the same initial conditions and the same sampling rate are also shown for the purpose of performance comparison. In Figure 4.2, the same initial conditions of the inverted pendulum are set for the simulation, and a uniformly distributed random noise with 0 mean and 0.083 variance is added to the system.

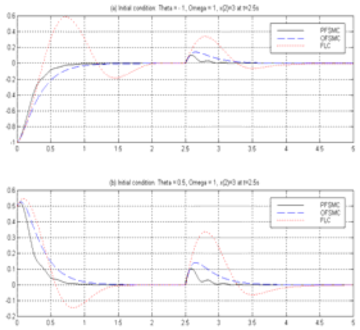

Figure 4.1 Simulation results without noise

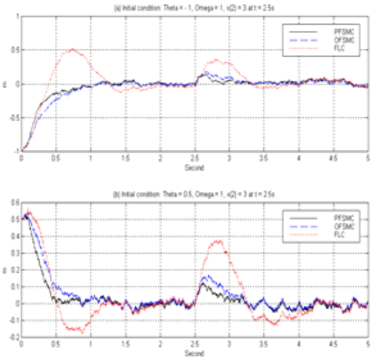

Figure 4.2 Simulation results with noise

5. Conclusions

A fuzzy sliding mode controller is designed in this paper by introducing sliding mode control into fuzzy logic control for discrete nonlinear systems with uncertainty, both the necessary and sufficient reaching condition of the sliding mode control are integrated into the controller. The proposed FSMC overcomes the chattering problem, and speed up the convergence compared with the original FSMC. The controller does not need the accurate system mathematical model, so it is relatively easy to design. The simulation results verify that the proposed controller superior to the original FSMC and the classical fuzzy logic controller in stability, convergence and robustness.

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